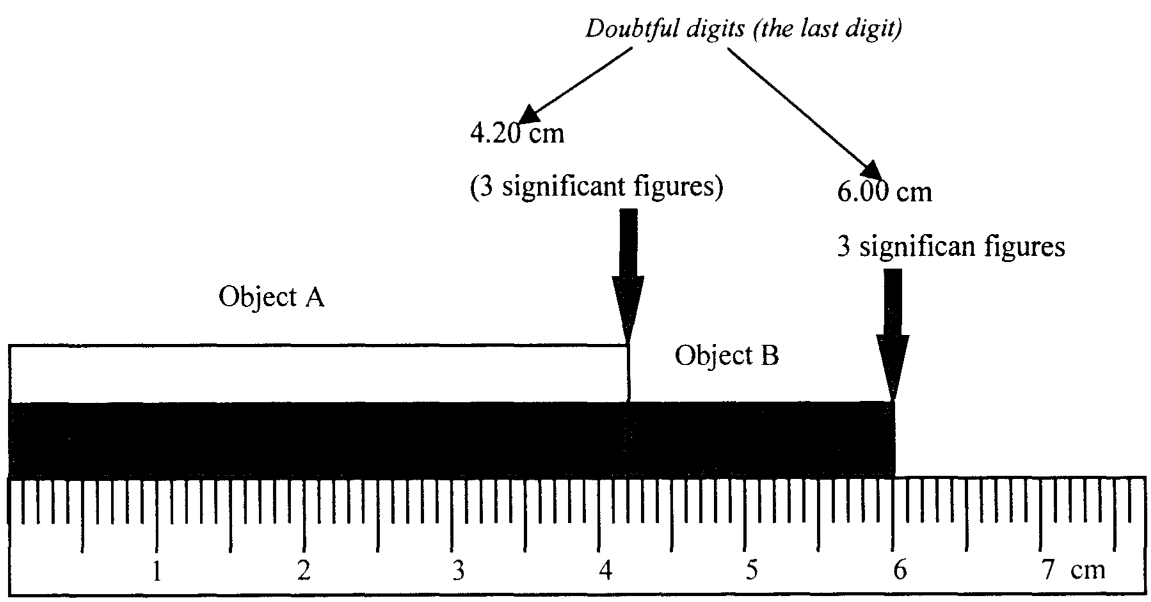
# I. SIGNIFICANT FIGURES

A typical measurement contains some ***certain digits*** plus a doubtful digit, which is the last ***doubtful figure***. All of the digits that are measured are significant figures. ***Significant figures*** include the certain and estimated digits in a measurement. Because the significant figures are measured digits, the measurements that contain more significant figures are necessarily more ***precise***[1] than measurements with fewer significant figures. This is why measured digits are called "significant figures." In order to honestly indicate reliability of an experimental work, it is important to maintain a proper number of significant figures in every measurement.

Use the following rules for determining which digits in a measurement are ***significant***:

1. ***Every nonzero digit is significant.***
2. ***Zeros appearing between nonzero digits are significant.***
3. ***All zeros to the right of the last nonzero digit are significant, only if the measurement contains a decimal point.***
4. ***Zeros appearing in front of the first nonzero digit are NOT significant. Zeroes that merely indicate magnitude are NOT significant.***
5. ***Zeros placed after the last nonzero digit are confusing when there is no decimal point used in a measurement. Use scientific notation to avoid the confusion.***

***Examples, Section I. SIGNIFICANT FIGURES***




Enlarged view of a "cm" ruler

[1] Precision vs. Accuracy: Accuracy indicates how close a value is to a true while precision refers to how close measurements are to each other. In contrast to accuracy, precision shows how close the measurement is to the exact reading using the given instrument. A very precise value with many significant figures is not necessarily accurate, because the instrument may be miscalibrated. However, the precise measurement is always helpful in increasing accuracy.

***Application of Rule 1.***

345 cm : 3 significant figures
49.56 : 4 significant figures

***Application of Rules 2 & 1.***

205 cm : 3 significant figures
40.05 cm : 4 significant figures

***Application of Rules 3 and others.***

430.0 cm : 4 significant figures
3.50 cm : 3 significant figures
40.50 cm : 4 significant figures

***Application of Rules 4 and others.***

0.004 g : 1 significant figures
0.0040 g : 2 significant figures
0.00405 g : 3 significant figures
0.004050 g : 4 significant figures

***Application of Rules 5 & 1.***

450 g : 2 or 3 significant figures (At least 2 significant figures )
4500 g : 2, 3 or 4 significant figures (At least 2 significant figures)

To avoid ambiguity created by the terminal zeros without any decimal point, use a scientific notation. ***All of the digits including zeros in the decimal number preceding the powers of ten are significant.***

The term "***significant***" used in science has a different meaning from that of the term "important." Zeros that are not significant (as in 0.001g) always indicates the magnitude and therefore they are important but they are not necessarily significant. However, zeros that are significant (as in 9.0 g) are not always important in indicating the magnitude. The term ***"significant"*** is related to the precision.

4500 g (2, 3, 4 significant figures) ↕

*The zeros in this notation are NOT known whether they are merely indicating the magnitude, or both magnitude and precision. Therefore, they are ambiguous.*

as 4.**500** x $10^3$ g (4 significant figures) ↕

*With or without these terminal zeros the magnitudes are NOT affected. Clearly, the zeroes in these notations are not related to the magnitude, but are associated with the precision. Therefore, they are significant.*

***When rounding off 5, raise or reduce it to the nearest even-number at one-place higher digit.*** See Practice B for examples.

There are two situations:

1. The measurement with a decimal point:
   For example, 4.50E5 g, 0.0550 g and 35.0 g contain 3 significant figures each, because
   ***when there is a decimal point, the first non-zero digit and all the following digits including zeroes are significant.***
2. The measurement without a decimal point:
   For example, 304000 g has 3 significant figures, because
   ***when there is no decimal point, all the digits from the first digit to, and including, the last non-zero digit are significant.***
   However, the zeroes after the last non-zero digit are unknown whether or not they are measured digits, because there is no decimal point.

***Application of Rule 5.***

Given a measurement, 450 g and 4500 g, if all of the recorded digits are significant, then they be recorded as follow:

450 g (2 or 3 significant figures) as 4.50 x $10^2$ g (3 significant figures)
4500 g (2, 3, 4 significant figures) as 4.500 x $10^3$ g (4 significant figures)

***Practice A:*** *Give the number of significant figures for each.*

1. 0.0333 g
2. 0.0303 g
3. 0.03030 g
4. 240.0 cm
5. 204.00 cm
6. 200.00 cm
7. 5.60E5 m
8. 5.600E-5 m
9. 8090 kg
10. 80900 kg
11. 0.500 dm
12. 0.505 dam
13. 0.0505 km
14. 0.05050 km
15. 50 boys and girls
16. 50 s
17. 501 s
18. 510 s
19. 0.050 h
20. 0.0505 m/s

| Answers | |
|---|---|
| 5. | 5 |
| 4. | 4 |
| 3. | 4 |
| 2. | 3 |
| 1. | 3 |
| 15. | Infinite because it is a count |
| 14. | 4 |
| 13. | 3 |
| 12. | 3 |
| 11. | 3 |
| 20. | 3 |
| 19. | 2 |
| 18. | 2 or 3 |
| 17. | 3 |
| 16. | 1 or 2 |
| 6. | 5 |
| 7. | 3 |
| 8. | 4 |
| 9. | 3 or 4 |
| 10. | 3, 4, or 5 |

***Practice B***. Re-express the following to retain 3 significant figures.

1. 50050 g
2. 5051 g
3. 5055 g
4. 5065 g
5. 5054 g
6. 0.00505555 g

*Answers*

| | |
|---|---|
| 5. | 5.05E3 g |
| 4. | 5.06E3 g |
| 3. | 5.06E3 g |
| 2. | 5.05E3 g |
| 1. | 5.00E4 g |
| 6. | 5.06E-3 g |

# II. ROUNDING OFF

When calculations are made using more than one measurements, the resulting values must be properly rounded off to make them consistent with input data they represent in regard to the precision. This consistency is important in showing the level of reliability and validity of data. There are three distinctive rules:

***1. In adding and subtracting, the answer should be rounded to the highest doubtful digit.***

***2. In multiplying and dividing, the answer should be rounded to the least number of significant figures among the input measurements.***[2]

***3. Counted whole numbers or defined rational numbers are exact and therefore they do not contain any doubtful digit. Because these numbers have an infinite number of significant figures, they do not restrict the number of digits to which calculated answers must to rounded off.***

[2] This is not a perfect rule but a simple one that is widely accepted at the secondary school level.

*Example 1. Rounding off the Sum*

The highest doubtful digit
The highest doubtful digit

Round off to the next higher digit

206.*2* cm
+40.5*67* cm
246.8 cm

The highest doubtful digit

35.69 g
- 5.*7* g
30.*0* g

*Example 2. Rounding off Product and Quotients*

3 significant figures

7.55 m * 0.34 m = 2.567 $m^2$ → Rounded to 2.6 $m^2$

2 significant figures,
the least number of significant figures

0.365 $m^2$ / 0.02000 m = 18.2*5* m → Rounded to 18.2 m

4 significant figures

3 significant figures, the least number of significant figures

5 is rounded to the nearest even number in the next higher digit.

*Example 3. Rounding off a Product Involving a Defined Number*

V = (4/3) π (10.0 cm)$^3$ = 4.19 x $10^3$ $cm^3$

3 significant figures

Not estimated but defined number,
an exact number without any doubtful digit,
and has an infinite number of significant figures.

# III. SCIENTIFIC NOTATION

There are three situations in which scientific notations are either necessary or convenient:

1. When zeros in a measurement without a decimal point create confusion,
2. When a measurement contains extremely large number equal to or more than 10, and
3. When a measurement contains extremely small number less than one.

In using the scientific notation, you must observe the following rule:

[*A Decimal Number*] x $10^{[Exponent]}$

***Less than 10, and equal to or greater than 1.***
***Keep only one nonzero digit to the left of the decimal point.***
Or

*[A Decimal Number] E [Exponent]*

***Examples***

1. If **50600 g** is read to 4 significant figures, it should be recorded **as 5.060 x10$^4$ g**.

   *3, 4, or 5 significant figures* — *4 significant figures*

2. The number of molecules in 1 mole of molecular substance is:

   602,000,000,000,000,000,000,000 = 6.02 x $10^{23}$ = 6.02**E23**

   *[Expressed in **scientific notation**]*

3. The mass of a proton in International System equals:

   0.000 000 000 000 000 000 000 000 001 672 623 kg = **1**.672623**E-27** kg
   = **1**.672623x$10^{-27}$ kg

   ***[One nonzero digit to the left of decimal point]***

***Practice Problems:*** *Express each in scientific notation keeping 4 significant figures.*

| | |
|---|---|
| 1 | 981.96 cm/s$^2$ |
| 2 | 0.0051 s |
| 3 | 6,240,500,000 g |
| 4 | 0.00050000 m |
| 5 | 0.00049995 m |

***Answers***

| | |
|---|---|
| 1 | 9.820E2 cm/s$^2$ |
| 4 | 5.000E-4 m |
| 3 | 6.240E9 g |
| 5 | 5.000E-4 m |
| 2 | 5.100E-3 s |

# IV. EXPONENTIAL ARITHMETIC

One of the most important calculation skills in science and mathematics is calculations using numbers written in powers-of-ten or scientific notation as presented in the previous section. Study the following six rules:

1. To add or subtract numbers written in scientific notation, they must first be expressed in the same power of ten before calculation.

ex. $\mathbf{5x10^4 + 2.0x10^5}$ = 5E4 + 2.0E5 = **0.5E5** + 2E5 = 2.5E5

2. To multiply powers of ten together, add their exponents.

| $\mathbf{(10^m)(10^n) = 10^{m+n}}$ or **(*E*m)(*E*n) = *E*(m+n)** |
|---|

ex. $(10^{-6})(10^4) = 10^{-6+4} = 10^{-2}$
or (E-6)(E4) = E(-6+4) = E-2

3. To multiply numbers expressed in scientific notation, multiply the decimal parts of the numbers together and add the exponents.

| $\mathbf{(A \times 10^m)(B \times 10^n) = (A \times B) \times 10^{(m+n)}}$ or **(A*E*m)(B*E*n) = (A*B)*E*(m+n)** |
|---|

ex. $(8.0 \times 10^{-5})(4.0 \times 10^8) = (8.0 \times 4.0) \times 10^{-5+8} = 32 \times 10^3 = 3.2 \times 10^4$
(8.0E-5)(4.0E8) = (8.0 x 4.0)E(-5+8) = 32E3 = 3.2E4

4. To divide one power of ten by another, subtract the exponent of the denominator from the exponent of the numerator.

| $\frac{10^m}{10^n} = 10^{m-n}$ or ***E*m/*E*n = *E*(m-n)** |
|---|

ex. $\frac{10^{-4}}{10^3} = 10^{(-4-3)} = 10^{-7}$ or (E-4)/E3 = E(-4-3) = E-7

5. To divide a number in scientific notation by another number in scientific notation, divide the decimal parts of the numbers and use Rule #4 to find the exponent of the quotient.

| $\frac{Ax10^m}{Bx10^n} = (\frac{A}{B})x10^{(m-n)}$ or **(A*E*m)/B*E*n) = (A*B)*E*(m-n)** |
|---|

ex. $\frac{8X10^4}{4X10^8} = (\frac{8}{4})x10^{(4-8)}$ $=2 \times 10^{-4}$ or (8E4)/(4E8) = (8/4)E(4-8) = 2E-4

6. To find the reciprocal of a power of ten, change the sign of the exponent.

| $\frac{1}{10^m} = 10^{-m}$ or **1/*E*m = *E*-m** |
|---|

ex. 1/E5 = E-5

*Practice Problems*

***Perform the following computations and express your answer in scientific notation:***

1. $8.00 \times 10^4$ m + $2.0 \times 10^3$ m =
2. $5.00 \times 10^{-3}$ cm + $4.00 \times 10^{-2}$ cm =
3. 2.00E5 g - 1.00E4 g =
4. 4.00E2 g + 3.000E4 g = ____________ g
5. $(10^3)(10^5)$ =
6. $(10^{-5})(10^5)$ =
7. $(10^{-3})(10^{-5})$ =
8. $(10^6)/(10^4)$ =
9. $(10^4)/(10^{-4})$ =
10. $(10^{-4})/(10^{-6})$ =
11. $(4.0 \times 10^4$ cm$)(2.00 \times 10^2$ cm$)$ =
12. (6.00E-3cm)(5.0E7cm) =
13. (5.0000E-5m)(2.000E3m)=
14. $(4.0 \times 10^4$ cm$)/(2.00 \times 10^2$ cm$)$ =
15. (6.00E-3cm)/(5.0E7cm) =
16. (5.0000E-5m)/(2.000E3m)=
17. Explosive liquid: 0.0006 kg/1000 ml = ____ g/ml
18. $1/10^5$ =
19. $1/10^{-5}$ =
20. ***White liquid:*** $\dfrac{(40080g) * 50}{(200.0cm)(156.0cm)(80.0cm)} =$

***Answers (assorted)***

| | |
|---|---|
| 6 | $10^0$ or 1 |
| 7 | $10^{-8}$ |
| 8 | $10^2$ |
| 9 | $10^8$ |
| 10 | $10^2$ |
| 1 | 8.20E4 m |
| 2 | ~~4.50E-2 cm~~ |
| 3 | 1.90E5 g |
| 4 | 3.000E4 g |
| 5 | $10^8$ |
| 11 | 8.0E6 $cm^2$ |
| 12 | 3.0E5 $cm^2$ |
| 13 | 1.000E-1 $m^2$ |
| 14 | 2.0E2 $cm^2$ |
| 15 | 3.0E-10 ~~cm~~ |
| 16 | 2.500E-8 $m^2$ |
| 18 | $10^{-5}$ |
| 19 | $10^5$ |
| 20 | 0.803 $g/cm^3$<br>It must be alcohol! |
| 17 | 0.6 g/ml<br>It must be gasoline! |

ok 3.000E4g + student has wrong answer

## V. METRIC SYSTEM

The standards of measurement used by scientists are those of the metric system. All the units of the metric system are based on 10 or multiples of 10. As a result, conversions between units are simple and easy. The metric system was originally established in France in 1790. The revised version of the metric system, the International System of Units (abbreviated ***SI***, after the French name "Le Systeme International d'Unites), was adopted by international agreement in 1960. The SI has the following seven base units:

| Base Quantity | Unit | SI Symbol |
|---|---|---|
| ***Length*** | ***meter*** | ***m*** |
| ***Mass*** | ***kilogram*** | ***kg*** |
| ***Time*** | ***second*** | ***s*** |
| *Electric current* | *ampere* | *A* |
| *Thermodynamic temperature* | *Kelvin* | *K* |
| *Amount of substance* | *mole* | *mol* |
| *Luminous intensity* | *candela* | *cd* |

The most commonly used SI prefixes that you should memorize are:

| *Factor* | *Prefix* | *Symbol* |
|---|---|---|
| **E9 ($10^9$)** | **giga** | **G** |
| **E6** | **mega** | **M** |
| **E3** | **kilo** | **k** |
| **E-1 ($10^{-1}$)** | **deci** | **d** |
| **E-2** | **centi** | **c** |
| **E-3** | **milli** | **m** |
| **E-6** | **micro** | **μ** |
| **E-9** | **nano** | **n** |

[Symbols are *capital character sensitive*, and therefore you should be careful not to change the symbols from capital to lower case characters, or vise versa.]

*Examples*

***It is very important for you to memorize and use the following metric units:***

| ***Length*** | | | ***Mass*** | | |
|---|---|---|---|---|---|
| ***Magnitude*** | ***Symbol*** | ***Name*** | ***Magnitude*** | ***Symbol*** | ***Name*** |
| E9m | ***Gm*** | *gigameter* | E9g | ***Gg*** | *gigagram* |
| E6m | ***Mm*** | *megameter* | E6g | ***Mg*** | *megagram* |
| E3m | ***km*** | *kilometer* | E3g | ***kg*** | *kilogram* |
| m | ***m*** | *meter* | g | ***g*** | *gram* |
| E-1m | ***dm*** | *decimeter* | E-1g | ***dg*** | *decigram* |
| E-2m | ***cm*** | *centimeter* | E-2g | ***cg*** | *centigram* |
| E-3m | ***mm*** | *millimeter* | E-3g | ***mg*** | *milligram* |
| E-6m | μm | *micrometer* | E-6g | μg | *microgram* |
| E-9m | ***nm*** | *nanometer* | E-9g | ***ng*** | *nanogram* |

***Example: Conversion Between Metric Units***

400 000 000 g = 400 Mg

*[Because the "Mg" unit is 1 000 000 times greater than the "g" unit, the numerical portion preceding "Mg" should be 1 000 000 times smaller than that preceding "g".]*

0.000 000 000 1 m = 0.1 nm

*[Because the "nm" unit is 1 000 000 000 times smaller than the "m" unit, the number for "nm" should be 1 000 000 000 times greater than that for "m".]*

***Always, remember that the smaller unit gets the larger number and the greater unit gets the smaller number to make the conversion to maintain equality. The ratio of two numbers can be determined by the factors of prefixes.***

# VI. ORGANIZING DATA

## A. Dependent and Independent Variables

In any scientific investigation, it is important to organize data in a table. The most important consideration in making a data table is to decide which are the independent and dependent variables. Examine the following description:

1. ***Independent variable***: This is the variable (a quantity that tends to change) that can affect the resulting data. Independent variable is usually controlled or arbitrarily selected by the experimenter.

2. ***Dependent variable***: This is the variable that depends on the independent variable. Usually the dependent variable is affected by the independent variable and is the subject of investigation.

## B. Titles

Everything you present in a report needs a title. The data table is an important part of your report, which requires a clearly stated title. In making a title of a table or graph, it is important to put the name of dependent variable first followed by the relationship and the name of the independent variable. In the terminology of mathematics, the table or graph displays *the dependent variable as a function of the independent variable.* Therefore, your title must name:

***The Dependent Variable as a Function of Independent Variable.***

## C. Tabulation of Data

The headings placed at the top of data columns usually give two items of information:

1. ***The name of the variable***
2. ***The unit of measure on which the values are based on.***

It is totally unnecessary to repeat writing the unit of measure with each numerical datum, because the unit is already included in the column heading.

In tabulating the data, it is important to keep ***the independent variable in the first column and the dependent variable in the next, or in the last column,*** when there are another quantities needed to compute the quantity of the dependent variable.

*Examples*

Table V. ***Distance as a Function of Time***

| Time (s) | Distance (cm) |
|---|---|
| 0.00 | 0.00 |
| 1.00 | 2.22 |
| 2.00 | 4.00 |
| 3.00 | 6.05 |
| 4.00 | 8.44 |
| 5.00 | 10.00 |
| 6.00 | 11.90 |
| 7.00 | 14.00 |
| 8.00 | 16.00 |
| 9.00 | 19.00 |
| 10.00 | 20.50 |

In the example (Table V), the distance measured is based on, affected by, and resulted from, the time that is controlled by the experimenter. Therefore, the distance is the dependent variable and the time the independent variable in this investigation.

The title shows the "***Distance***" (dependent variable) first, followed by "***as a Function of***" (the relationship) the ***Time*** (the independent variable). Also, notice that the starting letter of each word except short prepositions is capitalized. The term "and" or "vs." does not clearly indicate the relationship between two variables and fails to identify the dependent variable. The ***dependent variable*** is in fact the subject or topic variable, and therefore it is important to clearly identify that variable. The units of measure are omitted from the title for simplicity.

The time is kept in the first column because it is the independent variable. It is inconceivable for the time to depend on the distance in an ordinary situation. (This does not, however, mean that time is always an independent variable.) The distance which depended on the time in this example, is a dependent variable, and therefore is kept in the second or the last column.

## VII. MAKING A GRAPH

Graph is the simplest visual representation of experimental data. Once a graph is drawn, it can often disclose the pattern of change.

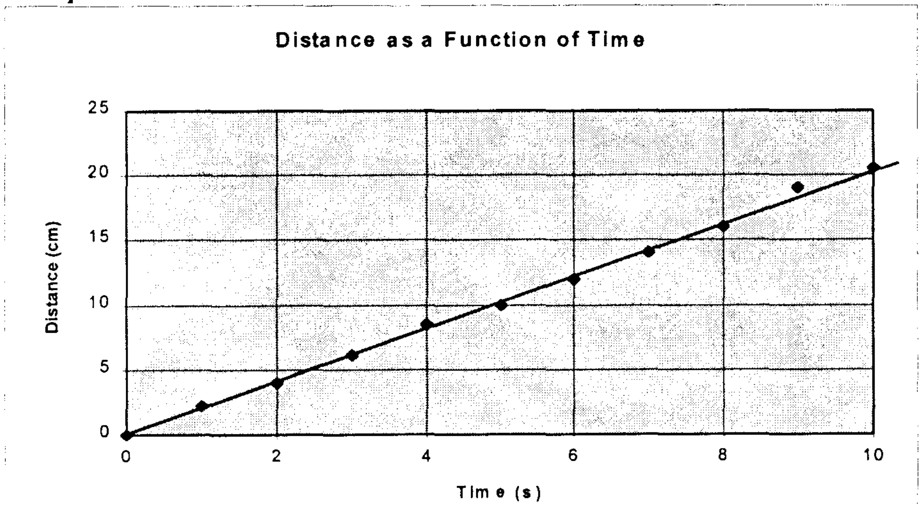
The graph also deserves a good title that identifies two variables and establishes their relationship. Follow the same rule used in making the title for a data table:

***Dependent Variable as a Function of Independent Variable.***

In order avoid confusion, it is important use a proper orientation of two axes:

***Always assign the dependent (y) variable to the vertical axis and the independent variable to the horizontal (x) axis. Also, it is imperative to include the units of measure in the labels of axes because, without the unit of measure, the number alone miserably fails to indicate the dimension.***

*Example*



# IX. ACCURACY AND PRECISION

*A. Accuracy.*

In order to show validity, it is important to analyze accuracy of an experimental result. One simple way to show the accuracy is to use a mathematical instrument called the percentage error using the following formula:

**Percentage Error** $= \frac{|A - E|}{A} * 100\%$

where A = the accepted value
E = the experimental value or the mean

This is sometimes called ***relative error*** because it is compared to the accepted value or the known standard value expressed in a ratio using percent. Note that 100 % = 1, and therefore it is necessary to keep the unit of "%" with "100" in the formula. The error in decimal or fractional ratio is converted into a percent value, and consequently the ratio value will increase by 100 but it will be in the unit of "%". ***Do not forget to multiply the decimal ratio by 100 and keep the "%" sign in the final result. And also, present this datum in your conclusion when you write your report.***

***B. Precision***

In order to indicate precision or randomness of experimental data, we can use the following mathematical instrument:

**Percentage Deviation from Mean** $= \frac{AverageDeviation}{Mean} * 100\%$

***A. Analyzing Accuracy***

Given A = 200.0 g
E = 180.0 g

**Percentage Error** $= \frac{|A - E|}{A} * 100\%$

$= \frac{|200.0 - 180.0|g}{200.0g} * 100\%$

= (20.0/200.0)*100%

= 10.0 %

### *B. Analyzing Precision*

Given a series of experimental data:

| Data | *Deviation from Mean* |
|---|---|
| 180.0 g | 0 g |
| 200.0 g | 20.0 g |
| 160.0 g | 20.0 g |
| 180.0 g | 0 g |
| ***Mean*** = 180.0 g | ***Average Deviation*** = 10.0 g |

Accepted Value = 200.0 g

$$\text{Percentage Deviation from Mean} = \frac{AverageDeviation}{Mean} * 100\%$$

= (10.0 g/180.0 g)*100%
=0.0556*100%
= 5.56%

***The accepted value (200.0 g) is not used but the amount of deviation is compared to its own mean value, thus indicating the degree of randomness of data.***

## X. Dimensional Analysis

In science, we frequently convert one dimension into another expressed in desired units of measure. This technique of conversion uses the simple concept of ratio between any two dimensions in question. Examine the following example where the dimension in g must be converted into another dimension in kg:

**506 g = _______ kg**

We know from metric system that,

1 kg = E3 g

Therefore,

$$\frac{E3g}{1kg} = 1 \qquad (1)$$

$$\frac{1kg}{E3g} = 1 \qquad (2)$$

This means that we can multiply one of these factors (equal to one) to any dimension without changing the original magnitude, if we want to. Let us return to our task, **506 g = _______ kg.** Let us try with the first conversion factor (1):

$$\textbf{? kg = 506 g*}\left(\frac{E3g}{1kg}\right) = (506*E3\ g*g/kg)\text{: } \textit{This is very messy!}$$

But if we use the second conversion factor (2),

$$\textbf{? kg = 506 g*}\left(\frac{1kg}{E3g}\right) = (506/E3)\ kg = \textbf{0.506 kg} \text{ : This works!}$$

The second conversion factor produced the desired dimension.  ***The key idea is to choose and multiply the conversion factor (ratio) that can cancel the unwanted unit while keeping the newly wanted unit in its place.*** Because this factor enables us to analyze dimensions toward conversion, this technique of conversion is called "***dimensional analysis***." ***This conversion factor is nothing more than a ratio between two dimensions***, usually equal or equivalent to each other. The real power of dimensional analysis lies in its simplicity (usually one step, without resorting to proportion equations) and that is why this technique is almost universally used in science, especially in chemistry.

***Dimensional Analysis***
***Practice Problems***

Convert each to a dimension in new units of measure. Show your dimensional analysis for each.

1 6.98E9 m = __________ km
2 0.9090 Mm = __________ m
3 0.00405 km = __________ cm
4 5.780E-5 kg = __________ g
5 40.0 $m^2$ = __________ $cm^2$
6 5.00E6 $cm^2$ = __________ $m^2$
7 708 $km^2$ = __________ $m^2$
8 8.09E8$cm^3$ = __________ $m^3$
9 0.0000609 $km^3$ = __________ $m^3$
10 4.24E6 $cm^3$ = __________ $m^3$
11 505 $cm^3$ = __________ $dm^3$
12 10.5 $dm^3$ = __________ $cm^3$
13 4.50 L = __________ $cm^3$
14 66.0 ml = __________ $dm^3$
15 60.0 miles = __________ km
16 *Speed limit on a highway*
=55 mph = __________ km/h
17 *Speed limit on a highway*
=90.0 km/h = __________ m/s
18 *Density of aluminum*
=2.69 g/$cm^3$ = __________ kg/$m^3$
19 *Density of ice*
=991 kg/$m^3$ = __________ g/$cm^3$
20 *Acceleration of a race car*
=55 mph in 6 s = __________ m/$s^2$

Answers

| | |
|---|---|
| 6 | 5.00E2 $m^2$ |
| 7 | 7.08E8 $m^2$ |
| 8 | 8.09E2 $m^3$ |
| 9 | 6.09E4 $m^3$ |
| 10 | 4.24 $m^3$ |
| 16 | 88 km/hr |
| 17 | 25.0 m/s |
| 18 | 2.69E3 kg/$m^3$ |
| 19 | 0.991 g/$cm^3$ |
| 20 | 4.1 m/$s^2$ |
| 11 | 0.505 $dm^3$ |
| 12 | 1.05E4 $cm^3$ |
| 13 | 4.50E3 $cm^3$ |
| 14 | 6.60E-2 $dm^3$ |
| 15 | 96.5 km |
| 1 | 6.98E6 km |
| 2 | 9.090E5 m |
| 3 | 4.05E2 m |
| 4 | 5.780E-2 g |
| 5 | 4.00E5 $cm^2$ |

## *The End*

# Measurement and Mathematical Skills

## MEASUREMENT

Science is built upon observation and measurement. Measurement involves a comparison of an unknown quantity with known, standard units. Measurements are always inexact because they are subject to error.

### Errors in Measurement.

Factors that affect the accuracy of measurements include flaws in the method used to obtain the measurement, fluctuations in the environment, limitations of the instruments used, and human error. Measurement errors fall into two basic groups—systematic and random.

**Systematic errors** tend to be in one direction, either too high or too low. For example, if a thermometer reads 5°C instead of 0°C in an ice and water mixture, then all of its readings will be too high by 5°C. Systematic error can be reduced by checking the method used and adjusting, or calibrating, all instruments.

**Random errors** tend to produce readings that fluctuate; some readings are too high, some are too low. Such fluctuations are always present in measurements. For example, repeated measurements of temperature of a liquid might produce readings of 56.4°C, 56.5°C, and 56.2°C. Random error can be reduced by taking the average of a large number of measurements and by controlling environmental fluctuations.

### Precision.

**Precision** refers to the smallest decimal place obtained by a measurement. Units are always specified when giving the precision of a measurement.

**Table 1**

| Measurement | Precision |
|---|---|
| 143 m | 1 m (the units place in meters) |
| 4.8 g | 0.1 g (the tenths place in grams) |
| 24.962 s | 0.001 s (the thousandths place in seconds) |

The scale markings on an instrument determine the possible precision of measurements made with that instrument. A measurement may be *estimated* to one-tenth of the smallest interval printed on the scale. Thus, the last recorded digit in a number obtained by a measurement is usually an estimate based on a reading between the smallest intervals marked on the scale. For example, the smallest printed interval on a 10-cm ruler is 1 mm, or 0.1 cm (Figure M-1). The possible precision when using this ruler is one-tenth of 1 mm, or 0.01 cm. Such a ruler could be used to obtain a reading of 7.68 cm, with the last digit (8) as an estimate. A reading of 7.683 cm would be beyond the possible precision of a measurement with this ruler.

### Significant Figures.

**Significant figures** are those digits that are obtained properly and directly from an instrument, including the final, estimated digit. In determining the significant figures in a measurement, keep in mind that any digit from 1 to 9 is always significant. The only digit that may not be significant is 0 since zeros are sometimes used to "hold place." Table 2 gives rules for determining which zeros in a measurement are significant and which are not significant.

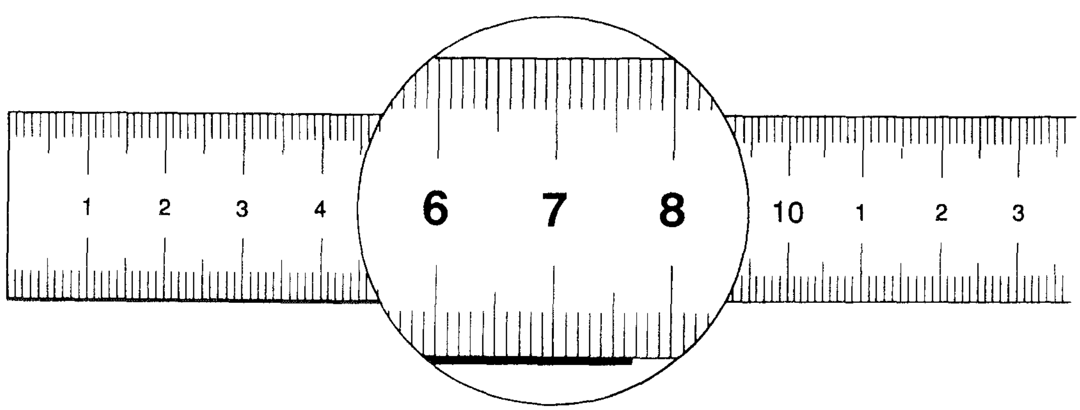



**Figure M-1.** Measuring the length of a line.

Table 2

| Rule | Example | Number of Significant Figures |
|---|---|---|
| 1. Zeros located at the end of a number and to the right of a decimal point are significant. They indicate the precision possible with the instrument used. | 3.0 g<br>12.3000 km<br>1.000 s<br>5.20 N | 2<br>6<br>4<br>3 |
| 2. Zeros located between significant digits are significant. | 30.9 V<br>402.06007 mm<br>1.030 ml | 3<br>8<br>4 (rules 1 and 2) |
| 3. Leading zeros are not significant. They may be included for clarity of format or to "hold place," but they are not the result of a measurement. | .042 kg<br>0.042 J<br>0.00000009 m<br>0.160 A<br>0.106 W<br>0.016 m/s<br>0.0010100 s | 2<br>2<br>1<br>3 (rules 1 & 3)<br>3 (rules 2 & 3)<br>2<br>5 (rules 1, 2, & 3) |
| 4. Zeros located at the end of a number and to the left of a decimal point are significant. | 40. °C<br>3000. K<br>250,600. g | 2<br>4<br>6 (rules 2 & 4) |
| 5. Zeros located at the end of a number are not significant if they are not followed by a decimal point. | 40 °C<br>3000 K<br>250,600 m | 1<br>1<br>4 (rules 2 & 5) |

**Accuracy.** The **accuracy** of a measurement refers to the agreement, or closeness, of its value to the true or accepted value. Accuracy may be expressed in terms of **absolute error** or **percent error**. In both cases, the absolute value of the difference between the measured and accepted values (indicated by vertical lines) is used to obtain a positive answer.

Absolute error

$$= |\text{Measured value} - \text{Accepted value}|$$

Percent error

$$= \frac{|\text{Measured value} - \text{Accepted value}| \times 100\%}{\text{Accepted value}}$$

## *Sample Problem*

The average of several measurements of the mass of an object is 48.60 g. Find the absolute error and percent error of this measurement if the actual mass of the object is 48.75 g.

*Solution:*

$$\text{Absolute error} = |48.60 \text{ g} - 48.75 \text{ g}| = 0.15 \text{ g}$$

$$\text{Percent error} = \frac{|48.60 \text{ g} - 48.75 \text{ g}| \times 100\%}{48.75 \text{ g}}$$

$$= 0.31\%$$

**Rounding Off in Calculations.** A chain is only as strong as its weakest link. Similarly, a calculated answer can only be as precise as the least precise measurement involved in the calculation. As a result, calculated answers must often be rounded off. If the digit to be dropped is less than 5, the digit to the left of it remains unchanged. For example, if 27.23 is rounded off to three significant figures, it becomes 27.2. If the digit to be dropped is 5 or more, the digit to the left of it is increased by 1; for example, 27.46 rounded off to three significant figures becomes 27.5.

In addition and subtraction, the answer must be rounded off to the same precision as the *least* precise number in the calculation. For example:

Addition:

$$\begin{array}{r} 6.12\ \text{g} \\ 18.3\ \ \ \text{g} \\ \underline{0.044\ \text{g}} \\ 24.464\ \text{g} \end{array} = 24.5\ \text{g}$$

In this calculation, 18.3g is the least precise number—to the tenths place.

Subtraction:

$$\begin{array}{r} 48.3639\ \text{m} \\ \underline{13.21\ \ \ \ \ \text{m}} \\ 35.1539\ \text{m} \end{array} = 35.15\ \text{m}$$

Here, 13.21 m is the least precise number—to the hundredths place.

In multiplication and division, the answer must be rounded off to contain the same number of significant figures as the measurement with the *least* number of significant figures.

# Significant Figures